Parametric processes of a strong laser in partially ionized plasmas

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The effects of the nonlinear polarization in a partially ionized plasma on the parametric processes of a strong laser are discussed. The nonlinear mode coupling equations and the linear growth rate of stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS) are derived. The numerical analyses for the nonlinear evolutions of SRS and SBS processes are given. Various effects of the second and the third order nonlinear susceptibilities on the SRS and SBS processes are discussed. The nonlinear evolutions of SRS and SBS processes are affected more efficiently than their linear growth rates by the nonlinear susceptibility.

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I. INTRODUCTION

Parametric processes in plasma are described by the interactions of an intense laser with plasma waves, supported by electrons and ions. For simplicity the ions are often assumed to be fully stripped, so that the electrons and ions are treated as free (unbound) charged particles. However, in intense laser-plasma interactions, e.g., in indirect laser fusion and x-ray lasers, the ions are typically only partially ionized. The bound electrons may be polarized by the laser (pump) wave and plasma waves, and reversibly affect the propagation of waves through the polarization term in the medium wave equations. Furthermore, when the wave intensity is high enough, the polarization of the partially ionized plasma is a nonlinear function of the wave field strength. Sprangle *et al.* [1,2] first indicated that bound electrons can cause two new kinds of unstable parametric processes: the atomic modulation instability (AMI) and atomic filamentation instability (AFI). In this paper we calculate the effects of bound electrons on the stimulated Raman scattering (SRS) and stimulated Brillouin scattering SBS processes. In laser-driven inertial confinement fusion (ICF), SRS and SBS are of crucial importance because they can induce significant losses of the incident laser energy and degrade the illumination symmetry required to reach a high implosion efficiency of the pellet. Previous works [1,2] have studied the effects of the linear and third nonlinear susceptibility on the AMI and AFI. Relatively little work has been reported on the effects of the linear, second, and third nonlinear susceptibilities on the SRS and SBS. This paper is organized as follows. Section II gives the basic equations for the laser wave (pump) and stimulated plasma waves, which includes scattering with the plasma electromagnetic wave, the plasma electrostatic wave, and the ion acoustic wave. The SRS process is discussed in Sec. III. The SBS process is discussed in Sec. IV. Conclusions are given in Sec. V, which indicate that the nonlinear susceptibility has affected the nonlinear evolutions of SRS and SBS processes efficiently.

II. BASIC EQUATIONS

The equations that describe the parametric processes in intense laser interactions with partially ionized plasma are as follows: the wave equations for the electromagnetic wave and electrostatic wave; the polarization of bound electrons in partially stripped ions; and the electron and ion fluid equations for the free electrons and ions. The general wave equation in a polarized medium is

$$\frac{\partial^2 \boldsymbol{E}}{\partial^2 t}+c^2[\nabla\times(\nabla\times\boldsymbol{E})]=-4\pi\frac{\partial \boldsymbol{J}}{\partial t}-4\pi\frac{\partial^2 \boldsymbol{P}}{\partial t^2}, \tag{1}$$

where $\boldsymbol{E}$ is the electric field of electromagnetic or electrostatic waves, $\boldsymbol{J}=-en_e\boldsymbol{v}_e$ is the free electron current, and $\boldsymbol{P}$ is the polarization due to the bound electrons. When the electric field of the wave $\boldsymbol{E}<\boldsymbol{E}_{ba}$ ($\boldsymbol{E}_{ba}$ is the electric field of the hydrogen atom at the Bohr radius, $|\boldsymbol{E}_{ba}|\sim 5.2\times 10^9$ V/cm, and the corresponding laser intensity $I_{ba}\sim 3.6\times 10^{16}$ W/cm^2), the nonlinear polarization of the medium can be expressed as a power series of $\boldsymbol{E}$ [3]:

$$\boldsymbol{P}\simeq\chi^{(1)}\boldsymbol{E}+\boldsymbol{\chi}^{(2)}\cdot\boldsymbol{E}\boldsymbol{E}+\chi^{(3)}(\boldsymbol{E}\cdot\boldsymbol{E})\boldsymbol{E}, \tag{2}$$

where $\chi^{(1)}$, $\chi^{(2)}$, and $\chi^{(3)}$ are the linear susceptibility, the second and third order nonlinear susceptibilities, respectively. For simplicity, we have taken $\chi^{(1)}$ and $\chi^{(3)}$ to be scalar quantities and $\chi^{(2)}$ to be a vector in Eq. (2).

The continuity and motion equations [4] of a free electron fluid are

$$\frac{\partial n}{\partial t}+\nabla\cdot(n\boldsymbol{v})=0, \tag{3}$$

$$\frac{\partial \boldsymbol{v}}{\partial t}+\frac{1}{2}\nabla(v^2)+\frac{e}{m}\boldsymbol{E}+\frac{\gamma_e k_B T}{mn}\nabla n=0. \tag{4}$$

Here n, $\boldsymbol{v}$, and γ_e are the density, velocity, and ratio of specific heats for an electron fluid, respectively; e and m are the charge and mass of an electron.

The equations of motion of an ion fluid are

$$\frac{\partial n_i}{\partial t}+\nabla\cdot(n_i\boldsymbol{v}_i)=0, \tag{5}$$

$$\frac{\partial \boldsymbol{v}_i}{\partial t}+\boldsymbol{v}_i\cdot\nabla(\boldsymbol{v}_i)+\frac{q}{m_i}\boldsymbol{E}_a+\frac{\gamma_i k_B T_i}{m_i n_i}\nabla n_i=0. \tag{6}$$

Here $q=Ze$ and m_i are the charge and mass of an ion, respectively, and $\boldsymbol{v}_i$ is the perturbed velocity of the ion fluid at low frequency. n_i is the density of the ion fluid, which is composed of the unperturbed and low frequency component: $n_i=n_{i0}+n_i^L$. E_a is the low frequency electric field produced by a separation of electrons and ions.

III. SRS PROCESS ($\omega_0=\omega_s+\omega_p$)

A. Coupling equations for SRS

For the SRS process, we have

$$\omega_0=\omega_s+\omega_p, \quad \boldsymbol{k}_0=\boldsymbol{k}_s+\boldsymbol{k}_p, \tag{7}$$

where 0 and s are the subscripts of the laser (pump) and scattering plasma electromagnetic wave, respectively, and p is the subscript of the electron plasma wave. Let

$$\boldsymbol{E}=\boldsymbol{E}_t+\boldsymbol{E}_p=\boldsymbol{E}_0+\boldsymbol{E}_s+\boldsymbol{E}_p,$$

$$\boldsymbol{v}=\boldsymbol{v}_t+\boldsymbol{v}_p=\boldsymbol{v}_0+\boldsymbol{v}_s+\boldsymbol{v}_p, \quad n=n_0(x)+n^h, \tag{8}$$

where $\boldsymbol{v}_t$ is the transverse velocity component, and n_0 and n^h are the background and perturbed high frequency density, respectively. From Eqs. (1)–(4), using $\nabla\cdot\boldsymbol{E}_t\equiv 0,\nabla\times\boldsymbol{E}_p\equiv 0,\nabla\cdot\boldsymbol{v}_t\equiv 0,\nabla\times\boldsymbol{v}_p=0$, yields the wave equations for SRS process:

$$\left(\frac{\partial^2}{\partial t^2}-\frac{c^2}{N_0^2}\nabla^2+\frac{\omega_{pe}^2}{N_0^2}\right)\boldsymbol{E}_t=\frac{4\pi}{N_0^2}\dot{\boldsymbol{j}}_{non}^t-\frac{4\pi}{N_0^2}\boldsymbol{\chi}^{(2)}\cdot\frac{\partial^2(\boldsymbol{E}\boldsymbol{E})_t}{\partial t^2}-\frac{4\pi}{N_0^2}\chi^{(3)}\frac{\partial^2[(\boldsymbol{E}\cdot\boldsymbol{E})\boldsymbol{E}]_t}{\partial t^2}, \tag{9}$$

$$\left(\frac{\partial^2}{\partial t^2}-\frac{3v_{th}^2}{N_0^2}\nabla^2+\frac{\omega_{pe}^2}{N_0^2}\right)\boldsymbol{E}_p=\frac{4\pi}{N_0^2}\dot{\boldsymbol{j}}_{non}^p-\frac{4\pi}{N_0^2}\boldsymbol{\chi}^{(2)}\cdot\frac{\partial^2(\boldsymbol{E}\boldsymbol{E})_p}{\partial t^2}-\frac{4\pi}{N_0^2}\chi^{(3)}\frac{\partial^2[(\boldsymbol{E}\cdot\boldsymbol{E})\boldsymbol{E}]_p}{\partial t^2}, \tag{10}$$

where $v_{th}\equiv(k_BT_e/m_e)^{1/2}$ and $N_0\equiv(1+4\pi\chi^{(1)})^{1/2}$. $\dot{\boldsymbol{j}}_{non}^t$ and $\dot{\boldsymbol{j}}_{non}^p$ are the rates of the nonlinear induced current [5] given by

$$\dot{\boldsymbol{j}}_{non}^t=-e\boldsymbol{v}_t[n\nabla\cdot\boldsymbol{v}_p+(\boldsymbol{v}_p\cdot\nabla+\boldsymbol{v}_t\cdot\nabla)n]-\frac{1}{2}en\nabla_t(\boldsymbol{v}^2)-\frac{\omega_{pe}^2}{4\pi n_0}n^h\boldsymbol{E}_t-e\frac{3k_BT}{m}\nabla_t n, \tag{11}$$

$$\dot{\boldsymbol{j}}_{non}^p=-en\boldsymbol{v}_p(\nabla\cdot\boldsymbol{v}_p)-e\boldsymbol{v}_p(\boldsymbol{v}\cdot\nabla)n-\frac{1}{2}en\nabla_p(\boldsymbol{v}^2)-\frac{\omega_{pe}^2}{4\pi n_0}n^h\boldsymbol{E}_p. \tag{12}$$

Let

$$\boldsymbol{E}_i(r,t)=\frac{1}{2}\boldsymbol{e}_i[\varepsilon_i e^{\mathrm{i}(\boldsymbol{k}_i\cdot\boldsymbol{r}-\omega_i t)}+\varepsilon_i^* e^{-\mathrm{i}(\boldsymbol{k}_i\cdot\boldsymbol{r}-\omega_i t)}], \tag{13}$$

$$\boldsymbol{v}_i(r,t)=\frac{1}{2}\frac{e}{\mathrm{i}m\omega_i}\boldsymbol{e}_i[\varepsilon_i e^{\mathrm{i}(\boldsymbol{k}_i\cdot\boldsymbol{r}-\omega_i t)}-\varepsilon_i^* e^{-\mathrm{i}(\boldsymbol{k}_i\cdot\boldsymbol{r}-\omega_i t)}], \tag{14}$$

where the subscript $i=0$, s, and p, respectively. The ε_i is the amplitude of $\boldsymbol{E}_i$ slowly varying in time and space, and $\boldsymbol{e}_i$ is the direction (polarization) vector of the electric field. Considering that the disturbance of the electron density with high frequency obeys the Poisson equation $n^h=-(1/4\pi e)\nabla\cdot\boldsymbol{E}_p$, and using Eqs. (11) and (12) in the wave equations, we obtain the mode coupling equations for the slowly varying electric field amplitude ε_i [6,7] (in the remainder of this paper, $i,j,k=0,s,p$, respectively, and when the i is determined, $j\neq i$, then $k\neq j,i$)

$$\left[(1+\delta_i)\frac{\partial}{\partial t}-\mathrm{i}\left(\Omega_i+\frac{\omega_i}{2}\delta_i\right)\right]\varepsilon_i=\left[A_i-\frac{2\pi\zeta_i}{N_0^2}\left(\frac{\partial}{\partial t}-\mathrm{i}\frac{\omega_i}{2}\right)\right]\varepsilon_{jk}-\varepsilon_i\frac{\partial\delta_i}{\partial t}, \tag{15}$$

$$\delta_i=\frac{2\pi\chi^{(3)}}{N_0^2}\left[\frac{3}{2}|\varepsilon_i|^2+\sum_{j\neq i}^{0,s,p}(1+2a_{ij}^2)|\varepsilon_j|^2\right],$$

$$a_{ij}=(\boldsymbol{e}_i\cdot\boldsymbol{e}_j), \quad \Omega_i=\frac{\omega_i^2-\omega_{i0}^2}{2\omega_i}, \quad \omega_{0,s}^2=\omega_{pe}^2+c^2k_{0,s}^2,$$

$$\omega_p^2=\omega_p^2+3v_{th}^2k_p^2, \quad \zeta_i=\sum_{j\neq i,k\neq j,i}(\boldsymbol{e}_i\cdot\boldsymbol{e}_j)(\chi^{(2)}\cdot\boldsymbol{e}_k).$$

$$\varepsilon_{ij}=\begin{cases}\varepsilon_s\varepsilon_p, & i=0\\ \varepsilon_0\varepsilon_p^*, & i=s\\ \varepsilon_0\varepsilon_s^*, & i=p,\end{cases}$$

$$A_i=\begin{cases}-Aa_{0s}\left(1+\dfrac{\omega_s\omega_p}{\omega_{pe}^2}\right), & i=0\\ -Aa_{0s}\left(1-\dfrac{\omega_0\omega_p}{\omega_{pe}^2}\right), & i=s\\ Aa_{0s}, & i=p,\end{cases}$$

$$A=\frac{ek_p\omega_{pe}^2}{4m\omega_0\omega_s\omega_p}. \tag{16}$$

Here A_i comes from the free electrons in the plasma, and the other terms come from the nonlinear susceptibilities $(\chi^{(2)},\chi^{(3)})$ of the bound electrons. Let

$$\varepsilon_i = g_i e^{\mathrm{i}\Omega_i + \mathrm{i}\alpha_i}, \quad i=0,s,p, \tag{17}$$

and assume $\Omega_0=\Omega_s+\Omega_p$ yields a set of equations for the real amplitudes g_i and real phases α_i,

$$\begin{aligned}(1+\delta_i)\frac{\partial g_i}{\partial t} &= \left[A_i \cos\gamma + \frac{2\pi\zeta_i}{N_0}\left(\frac{\omega_i}{2}-\Omega_i\right)\sin\gamma_i\right] g_j g_k \\ &\quad - \frac{2\pi\zeta_i}{N_0}\left[\cos\gamma \frac{\partial g_j g_k}{\partial t} + \sin\gamma_i \frac{\partial \alpha_{jk}}{\partial t} g_j g_k\right] \\ &\quad - g_i \frac{\partial \delta_i}{\partial t},\end{aligned} \tag{18}$$

$$\begin{aligned}(1+\delta_i) g_i \frac{\partial \alpha_i}{\partial t} &= -\left[A_i \sin\gamma_i - \frac{2\pi\zeta_i}{N_0}\left(\frac{\omega_i}{2}-\Omega_i\right)\cos\gamma\right] g_j g_k \\ &\quad + \frac{2\pi\zeta_i}{N_0}\left[\sin\gamma_i \frac{\partial g_j g_k}{\partial t} - \cos\gamma \frac{\partial \alpha_{jk}}{\partial t} g_i g_k\right] \\ &\quad + \delta_i g_i\left(\frac{\omega_i}{2}-\Omega_i\right),\end{aligned} \tag{19}$$

where $\gamma=\alpha_0-\alpha_s-\alpha_p$, and

$$\alpha_{jk} = \begin{cases} \alpha_s+\alpha_p, & i=0 \\ \alpha_0-\alpha_p, & i=s \\ \alpha_0-\alpha_s, & i=p, \end{cases}$$

$$\gamma_i = \begin{cases} \gamma, & i=0 \\ -\gamma, & i=s \\ -\gamma, & i=p. \end{cases}$$

B. Linear analysis of SRS process

When $\chi^{(2)}=\chi^{(3)}=0$, the above equations will return to the conventional mode coupled equations of SRS in a fully stripped plasma [6]. Equations (18) and (19) are highly nonlinear, and usually can only be solved numerically (see the following text). To understand the features of SRS in partially stripped plasmas qualitatively, a linear analysis of the instability of SRS is given below. In the initial phase of SRS instability, we have

$$g_0(t) \simeq g_{00} = \text{const}, \quad g_s \sim g_p \sim 0 \ll g_{00},$$

$$\frac{\partial \alpha_i}{\partial t} \sim 0 (i=0,s,p), \quad \gamma = \text{const}.$$

Thus the evolution equations of g_0 and α_i need not be solved. After linearization, the evolution equations of g_s and g_p are

$$(1+\delta_s^0)\frac{\partial g_s}{\partial t} = A_s' g_p - \zeta_s' \frac{\partial g_p}{\partial t},$$

$$(1+\delta_p^0)\frac{\partial g_p}{\partial t} = A_s' g_s - \zeta_p' \frac{\partial g_s}{\partial t},$$

$$\delta_i^0 \simeq \frac{2\pi\chi_{(3)}}{N_0^2}(1+2a_{i0}^2)g_{00}^2, \quad \zeta_i' = \frac{2\pi\zeta_i}{N_0^2} g_{00}\cos\gamma,$$

$$A_i' = \left(A_i\cos\gamma - \frac{\pi\zeta_i}{N_0^2}\omega_i \sin\gamma\right) g_{00} \quad (i=s,p).$$

Let $g_i(t)=g_{i0}\exp(\Gamma t)(i=s,p)$, The growth rate of SRS instability can be derived from the linear algebraic equations

$$\begin{aligned}\Gamma &= \frac{1}{2[(1+\delta_s^0)(1+\delta_p^0)+\zeta_s'\zeta_p']}\cdot(-(\zeta_p' A_s' + \zeta_s' A_p') \pm \{(\zeta_p' A_s' \\ &\quad + \zeta_s' A_p')^2 + 4A_s' A_p' [(1+\delta_s^0)(1+\delta_p^0)+\zeta_s'\zeta_p']\}^{1/2}).\end{aligned} \tag{20}$$

When ignoring the nonlinear susceptibilities, we have

$$\delta_i = \delta_i^0 = \zeta_i = \zeta_i' = 0, \quad A_i' = A_i \cos\gamma g_{00} \quad (i=s,p).$$

Equation (20) then reduces to the linear instability rate of conventional SRS[5]:

$$\Gamma_0 = \sqrt{A_s A_p} g_{00} |\cos\gamma| \simeq \frac{k_p v_0}{4}\frac{\omega_p}{\omega_s}|\boldsymbol{e}_0\cdot\boldsymbol{e}_s||\cos\gamma|, \tag{21}$$

where the v_0 is quiver velocity. In deriving the last step of Eq. (21), Eq. (16) and $\omega_0 \sim 2\omega_p$, $\omega_p \sim \omega_{pe}$, $Ag_{00} = (k_p v_0/4)(\omega_p/\omega_s)$ were used.

When the linear terms of $\delta_i(\chi^{(3)})$ and $\zeta_i(\chi^{(2)})$ remain, Eq. (20) can be written as $\Gamma \simeq \Gamma_0 + \Gamma^{(2)} + \Gamma^{(3)}$, where $\Gamma^{(2)}$ and $\Gamma^{(3)}$ are the modifications of the growth rate due to the second and third order susceptibilities, respectively,

$$\Gamma_0 + \Gamma^{(3)} = \frac{\Gamma_0}{\sqrt{1+\delta_s^0+\delta_p^0}} \simeq \Gamma_0 - \frac{1}{2}(\delta_s^0+\delta_p^0)\Gamma_0, \tag{22}$$

$$\begin{aligned}\Gamma^{(3)} &\simeq -\frac{1}{2}(\delta_s^0+\delta_p^0)\Gamma_0 = -\frac{2\pi\chi^{(3)}}{N_0^2}(1+a_{0s}^2+a_{0p}^2)g_{00}^2 \\ &= -\frac{2N_2}{N_0^2}(1+a_{0s}^2+a_{0p}^2)I_0 \quad (a_{ij}=\boldsymbol{e}_i\cdot\boldsymbol{e}_j).\end{aligned} \tag{23}$$

The last expression is easier to compare with the observations, N_2 is the "effective nonlinear refractive index" [2], and I_0 is the pump wave intensity

$$N_2 = \frac{8\pi^2}{cN_0}\chi^{(3)}, \quad I_0 = \frac{c}{4\pi}N_0\langle \boldsymbol{E}_0\cdot\boldsymbol{E}_0\rangle = \frac{c}{8\pi}N_0 g_{00}^2.$$

From Eq. (22), we find that the third order susceptibility can reduce the growth rate Γ_0 of SRS by a factor $1/\sqrt{1+\delta_s^0+\delta_p^0}$.

The effect of the second order susceptibility on the linear growth rate of SRS is

$$\Gamma^{(2)} \simeq -\frac{\pi}{N_0^2}(\zeta_p A_s + \zeta_s A_p)\cos^2\gamma g_{00}^2 - \frac{\pi}{N_0^2}\frac{1}{2}\Gamma_0\left(\frac{\zeta_p\omega_p}{A_p} + \frac{\zeta_s\omega_s}{A_s}\right)\tan\gamma. \tag{24}$$

Because $\tan\gamma$, ζ_s, and ζ_p can all be positive or negative, the effects of the second order susceptibility on the linear growth rate of SRS are complex. On the other hand, the value of $\chi^{(2)}(\zeta_{s,p})$ may be very small in a partially stripped plasma ($\chi^{(2)}$ can exist only in those kinds of partially stripped ion, in which the electrostatic potential experienced by the bound electrons is not radially symmetric [3]). In the following, we shall briefly evaluate the value of $\Gamma^{(3)}$. According to our derivation,

$$\delta_i^0 = \frac{2\pi\chi^{(3)}}{N_0^2}(1+2a_{i0}^2)g_{00}^2, \quad N_0^2 = 1+4\pi\chi^{(1)},$$

$$a_{i0} = \boldsymbol{e}_i \cdot \boldsymbol{e}_0 \ (i=s,p), \tag{25}$$

where $\chi^{(1)}$ and $\chi^{(3)}$ are all proportional to the density of the partially stripped ions n_i. When the charge state is small compared to the atomic number,and the pump wave frequency is far below any atomic resonance, the values of the above two susceptibilities are given in Appendix A of Ref. [2] (where N_2 is expressed as η_2),

$$\chi^{(1)} \simeq 4\times10^{-24} n_a \ (\mathrm{cm}^{-3}), \quad \chi^{(3)} \sim 10^{-38} n_i \ (\mathrm{cm}^{-3}),$$

$$N_2(\mathrm{cm^2/W}) = \frac{12\pi^2}{c}\frac{\chi^{(3)}}{N_0^2}\times10^7 \sim 5\times10^{-40} n_i(\mathrm{cm}^{-3}).$$

In the plasma produced in the laser fusion and laser solid target interaction, where $n_e \sim 0.2 n_c$ (for $\lambda_0 \sim 300$ nm, $n_c \sim 1.2\times10^{22}$ cm^{-3}, n_c is the critical electron density), the ion density near the laser-interaction region is about $n_i \sim 10^{21}$ cm^{-3}, so we have

$$N_0 \sim 1 + 4\times10^{-3} \simeq 1,$$

$$N_2 \sim 5\times10^{-19}\ \mathrm{cm^2/W},$$

$$\delta_i^0 \sim 2(1+2a_{ij}^2)N_2 I_0 \sim 10^{-18} I_0 \ (\mathrm{W/cm^2}). \tag{26}$$

In the above formulas, the effect of $\chi^{(2)}$ is ignored. From Eqs. (22) and (26) we can obtain the following conclusions: If the frequency of the pump laser is far below any atomic resonance, the obvious effect of $\chi^{(3)}(N_2)$ on the linear growth rate of SRS occurs when $I_0 > 10^{17}$ W/cm^2.

C. The nonlinear evolutions of the SRS

Now, we discuss the complete solutions of Eqs. (18) and (19). Because $g_{0i}(t)$ and $\alpha_i(t)$ are functions of time, they can only be solved numerically. We define the following dimensionless quantities: $y_1 = g_0/g_{00}$, $y_2 = g_s/g_{00}$, $y_3 = g_p/g_{00}$, $y_4 = \alpha_0/10$, $y_5 = \alpha_s/10$, $y_6 = \alpha_p/10$, $y_7 = y_4 - y_5 - y_6$, and $\gamma = \alpha_0 - \alpha_s - \alpha_p$, $w = \omega_0 t$. The initial values are $g_{s0}/g_{00} = 0.05$, $g_{p0}/g_{00} = 0.05$, $\alpha_{00} = \alpha_{s0} = \alpha_{p0} = 0$, $T_e = 3$ keV, $n_i = 10^{21}$ cm^{-3}, $g_{00} = \sqrt{10^7 8\pi I_0/N_0}$, $\chi^{(1)}$(esu) $= 4\times10^{-24} n_i$, $\omega_0 = 6.3\times10^{15}$ s^{-1}, $\omega_s \simeq \omega_p \simeq \omega_0/2$, $\alpha_{0s} = \pi/4$, $\alpha_{sp} = \pi/2$, $\boldsymbol{\chi}^{(2)}\cdot\boldsymbol{e}_0 = \chi^{(2)}$, $\boldsymbol{\chi}^{(2)}\cdot\boldsymbol{e}_s = \chi^{(2)}\cos(\pi/4)$, and $\boldsymbol{\chi}^{(2)}\cdot\boldsymbol{e}_p = \chi^{(2)}\cos(\pi/4)$.

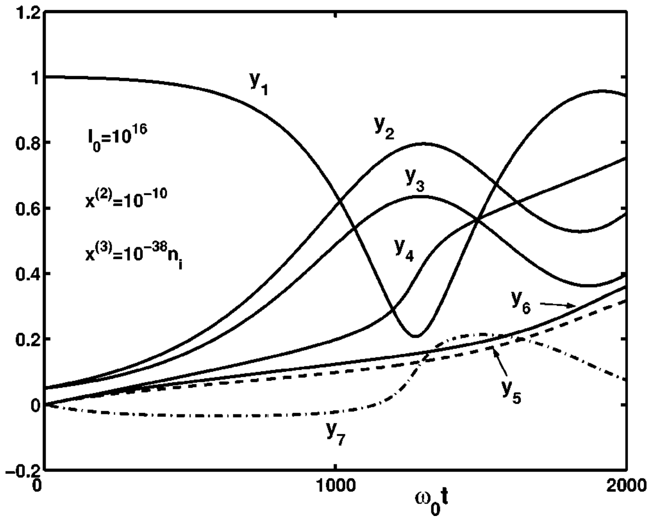

FIG. 1. General behavior of SRS nonlinear evolution with pump initial intensity $I_0 = 10^{16}$W/cm^2, $\chi^{(2)} = 10^{-10}$ esu, $\chi^{(3)} = 10^{-38} n_e$ esu. y_1, y_2, y_3 are the relative amplitudes (relative to the initial amplitude of the pump) of pump, scattering, plasma wave, respectively, and y_4, y_5, y_6, y_7 are corresponding relative phases and relative phases difference (relative to 10).

Figure 1 shows the general behavior of SRS nonlinear evolution. The y_2 and y_3 (amplitudes of scattering waves) rise quickly, reach a maximum (saturation), and then decay. At the same time, the injecting laser wave experiences a similar but opposite evolution process. Assuming no dissipation mechanism in our model, the evolution processes of all wave amplitudes are periodical. The time for the amplitude to reach its extreme value is referred to as the characteristic saturation time. While the evolutions of wave amplitudes are periodical, the evolutions of wave phases monotonously increase. When the amplitude evolutions pass through their extreme points, the pump wave phase y_4 and phase difference γ among three waves undergo a sudden change, while the scattering wave phases y_5, y_6 behave smoothly. This indicates that the sudden jump of the pump wave phase is due to the periodical behavior in the nonlinear evolution of the wave amplitude. In the very beginning, the amplitudes of unstable scattering waves arise with time exponentially, as shown in Fig. 2(a). This coincides with the linear instability analysis in the above. The effects of nonlinear susceptibilities on the SRS nonlinear evolutions are in follows.

(a) *Effects of the third order nonlinear susceptibility* $\chi^{(3)}(\chi^{(2)}=0)$. Figure 2(a) shows the effect of $\chi^{(3)}$ on the SRS while the pump strength (hence amplitude) is constant ($I_0 = 10^{16}$ W/cm^2). The growth rate of the scattering wave decreases with increasing $\chi^{(3)}$, but the scattering saturation time gets longer and the scattering saturation strength has

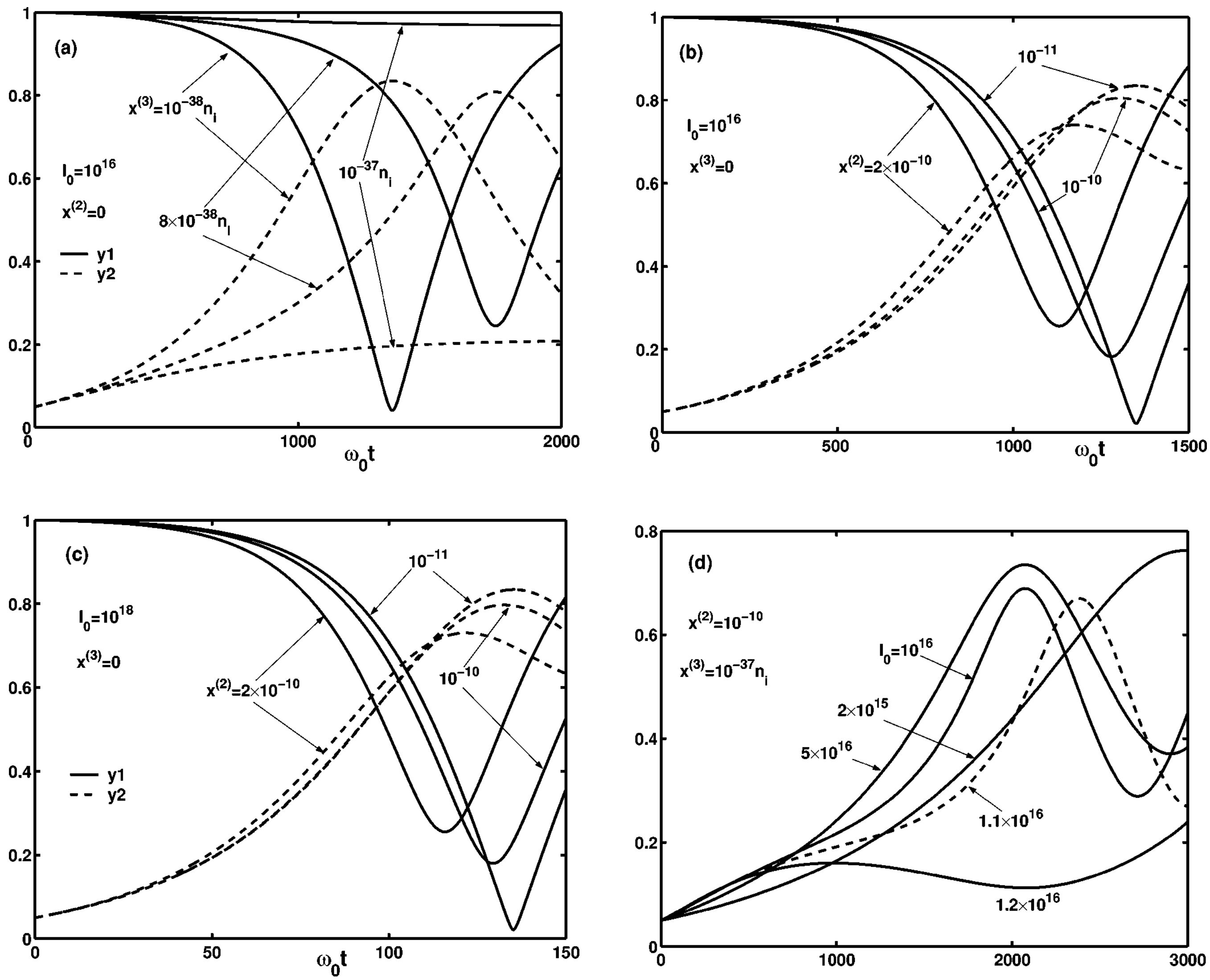

FIG. 2. (a)–(d) The relative amplitudes of the scattering electromagnetic wave evolves with time when two of I_0 (W/cm^2), $\chi^{(2)}$ (esu), and $\chi^{(3)}$ (esu) are constant and another is various in SRS.

almost no change. When $\chi^{(3)}=10^{(-37)}n_i$, the scattering strength becomes very small and can be nearly ignored while the pump strength has almost no attenuation [just as shown in Fig. 2(a)]. In Fig. 2(a) the strength of the pump wave is 10^{16} W/cm^2, and obvious differences occur in the SRS evolution process. This is different from the prediction of linear analysis, in which the obvious effects of the $\chi^{(3)}$ on the SRS linear growth rate occur only when $I_0 \geqslant 10^{17}$ W/cm^2. This confirms a conclusion of Ref. [6] that the effects of third order susceptibility $\chi^{(3)}$ on the SRS are more important in the nonlinear regime.

(b) *Effects of the second order nonlinear susceptibility* $\chi^{(2)}(\chi^{(3)}=0)$. Figure 2(b) shows the effect of $\chi^{(2)}$ on SRS evolution at $I_0=10^{16}$ W/cm^2. The most obvious effect of the $\chi^{(2)}$ on the SRS process is that the saturation amplitude of the scattering wave decreases with increasing $\chi^{(2)}$, while the growth rate (saturation time) increases (shortens) slightly. Figure 2(c) is similar to Fig. 2(b) except for $I_0 = 10^{18}$ W/cm^2. In Fig. 2(c), the saturation time (about 0.02 ps) is much less than the corresponding saturation time (about 0.2 ps) in Fig. 2(b). These show that the initial pump wave strength has enormous effects on the SRS processes. Comparing Figs. 2(a) and 2(b), we can find that the effect of $\chi^{(3)}$ on the growth rate of the scattering wave is more notable than that of $\chi^{(2)}$.

(c) *Effects of both second and third order nonlinear susceptibilities* $(\chi^{(2)},\chi^{(3)}\neq 0)$. The effects of both $\chi^{(2)}$ and $\chi^{(3)}$ on the SRS process are revealed in Fig. 2(d). When $\chi^{(2)}$ and $\chi^{(3)}$ are fixed, we observe the effects of pump wave strength I_0 on the SRS. With increasing I_0, the relative saturation amplitudes of the scattering wave monotonically decrease slightly, and the growth rate monotonically increases.The behaviors of saturation times are complex: there is a critical strength $I_{0c}\approx 5\times 10^{15}$ W/cm^2, which may be relevant to $\chi^{(3)}$. When $I<I_{0c}$ the saturation time monotonically decreases with increasing I_0, but when $I_0>I_{0c}$ the saturation time monotonically increases with increasing I_0.

IV. SBS PROCESS ($\omega_0=\omega_s+\omega_a$)

A. SBS coupling equations

For the SBS process, we have

$$\omega_0=\omega_s+\omega_a,\boldsymbol{k}_0=\boldsymbol{k}_s+\boldsymbol{k}_a,$$

where $0,s$ is the subscript of the laser (pump) and the scattering plasma electromagnetic wave, respectively, and a is the subscript of the ion acoustic wave. Let

$$\boldsymbol{E}=\boldsymbol{E}_t+\boldsymbol{E}_a=\boldsymbol{E}_0+\boldsymbol{E}_s+\boldsymbol{E}_a,$$

$$\boldsymbol{v}=\boldsymbol{v}_t+\boldsymbol{v}_a=\boldsymbol{v}_0+\boldsymbol{v}_s+\boldsymbol{v}_a,\quad n=n_0(x)+n^h+n^L,$$

where subscript $t(0$ or $s)$ means transversal wave, and n_0,n^h,n^L is the undisturbed electron density, disturbing electron density with high frequency and disturbing electron density with low frequency, respectively. From Eqs. (1)–(4), using $\boldsymbol{\nabla}\cdot\boldsymbol{E}_t\equiv 0$, $\boldsymbol{\nabla}\cdot\boldsymbol{v}_t\equiv 0$, $\boldsymbol{\nabla}\times\boldsymbol{v}_a=0$, yields the scattering wave equations for the SBS process [8]:

$$\left(\frac{\partial^2}{\partial t^2}-\frac{c^2}{N_0^2}\nabla^2+\frac{\omega_{pe}^2}{N_0^2}\right)\boldsymbol{E}_t=\frac{4\pi}{N_0^2}\dot{\boldsymbol{j}}^t_{non}-\frac{4\pi}{N_0^2}\boldsymbol{\chi}^{(2)}\cdot\frac{\partial^2(\boldsymbol{EE})_t}{\partial t^2}-\frac{4\pi}{N_0^2}\boldsymbol{\chi}^{(3)}\frac{\partial^2[(\boldsymbol{E}\cdot\boldsymbol{E})\boldsymbol{E}]_t}{\partial t^2},\quad (27)$$

$$\dot{\boldsymbol{j}}^t_{non}=-e\boldsymbol{v}_t[n(\nabla\cdot\boldsymbol{v}_a)+(\boldsymbol{v}\cdot\nabla)n]-en\nabla(v^2)-\frac{e^2}{m}(n^h+n^L)\boldsymbol{E}_t-\frac{e\gamma_e k_B T}{m}\nabla n,\quad (28)$$

$$\boldsymbol{E}_a=-\frac{m}{2e}\nabla(v^2)^L-\frac{\gamma_e k_B T}{en}\nabla n.\quad (29)$$

From Eqs. (5) and (6), using $\nabla\times\boldsymbol{v}_i=0$, we can obtain

$$\frac{\partial^2 n_i}{\partial t^2}-C_s^2\nabla^2 n_i=\nabla\cdot\left[\boldsymbol{v}_i\boldsymbol{v}_i\cdot\nabla n_i+n_i\boldsymbol{v}_i\nabla\cdot\boldsymbol{v}_i+\frac{1}{2}n_i\nabla\left(v_i^2+\frac{m_e Z}{m_i}(v^2)^L\right)\right],$$

$$C_s^2=\frac{k_B(Z\gamma_e T+\gamma_i T_i)}{m_i}.\quad (30)$$

Let

$$\boldsymbol{E}_i(\boldsymbol{r},t)=\frac{1}{2}\boldsymbol{e}_i[\varepsilon_i e^{\mathrm{i}(\boldsymbol{k}_i\cdot\boldsymbol{r}-\omega_i t)}+\varepsilon_i^* e^{-\mathrm{i}(\boldsymbol{k}_i\cdot\boldsymbol{r}-\omega_i t)}],\quad i=0,s,a,\quad (31)$$

and from Eq. (29), we have

$$\boldsymbol{E}_a=-\frac{m}{e}\nabla(\boldsymbol{v}_0\cdot\boldsymbol{v}_s)-\frac{\gamma_e k_B T}{en_{i0}}\nabla n^L.$$

Using

$$n^L=\frac{1}{2}[Ne^{\mathrm{i}(\boldsymbol{k}_a\cdot\boldsymbol{r}-\omega_a t)}+N^* e^{-\mathrm{i}(\boldsymbol{k}_a\cdot\boldsymbol{r}-\omega_a t)}],\quad (32)$$

and the expression (which is derived from the electron motion equation in the first order approximation)

$$\boldsymbol{v}_t(\boldsymbol{r},t)=\frac{1}{2}\boldsymbol{e}_t\frac{e}{\mathrm{i}m\omega_i}[\varepsilon_i e^{\mathrm{i}(\boldsymbol{k}_a\cdot\boldsymbol{r}-\omega_a t)}+\varepsilon_t^* e^{-\mathrm{i}(\boldsymbol{k}_a\cdot\boldsymbol{r}-\omega_a t)}],$$

$$t=0,s,\quad (33)$$

we obtain

$$\varepsilon_a=-\mathrm{i}(a_1\varepsilon_0\varepsilon_s^*+a_2 N),\quad a_1=\frac{k_a e}{2m\omega_0\omega_s},$$

$$a_2=\frac{k_a\gamma_e k_B T}{en_{i0}}\equiv\frac{4\pi e}{m}\frac{k_a}{\omega_{pe}^2}\gamma_e k_B T.\quad (34)$$

Using $n^h=-(1/4\pi e)\nabla\cdot\boldsymbol{E}_p$, and Eqs. (31), (33), and (34) in Eqs. (27) and (30), we can derive the mode coupled equations for the lowly varying electric amplitude ε_i ($i=0,s,a$):

$$\left[(1+\delta_i)\frac{\partial}{\partial t}-\mathrm{i}\left(\Omega_i+\frac{\omega_i}{2}\delta_i\right)\right]\varepsilon_i=\mathrm{i}B_i-\frac{2\pi\zeta_i}{N_0^2}\left(\frac{\partial}{\partial t}-\mathrm{i}\frac{\omega_i}{2}\right)\varepsilon_{jk}-\varepsilon_i\frac{\partial\delta_i}{\partial t},\quad (35)$$

$$\delta_i=\frac{2\pi\chi^{(3)}}{N_0^2}\left[\frac{3}{2}|\varepsilon_i|^2+\sum_{j\neq i}^{0,s,a}(1+2a_{ij}^2)|\varepsilon_j|^2\right],$$

$$a_{ij}=(\boldsymbol{e}_i\cdot\boldsymbol{e}_j),\quad i=0,s,\quad \delta_a\equiv 0,$$

$$\Omega_i=\frac{\omega_i^2-\omega_{i0}^2}{2\omega_i},\quad i=0,s,a,\quad \omega_{00,s0}^2=\frac{\omega_{pe}^2+c^2k_{0,s}^2}{N_0^2},$$

$$\omega_{a0}^2=k_a^2C_s^2,$$

$$\zeta_i(\boldsymbol{\chi}^{(2)})=\sum_{j\neq i,k\neq j,i}^{0,s,a}(\boldsymbol{e}_i\cdot\boldsymbol{e}_j)(\boldsymbol{\chi}^{(2)}\cdot\boldsymbol{e}_k),\quad i=0,s,\quad \zeta_a\equiv 0,$$

$$\varepsilon_{ij}=\begin{cases}\varepsilon_s\varepsilon_a, & i=0\\ \varepsilon_0\varepsilon_a^*, & i=s\\ 0, & i=a,\end{cases}$$

$$B_i=\begin{cases}\beta_0 N\varepsilon_s, & i=0\\ \beta_s N^*\varepsilon_0, & i=s\\ -\beta_a\varepsilon_0\varepsilon_s^*, & i=a,\end{cases}$$

$$\beta_0=-\frac{\omega_{pe}^2}{4n_0\omega_0}a_{0s},\quad \beta_s=-\frac{\omega_{pe}^2}{4n_0\omega_s}a_{0s},$$

$$\beta_a=\frac{k_a^2\omega_{pi}^2}{16\pi m\omega_0\omega_s\omega_a}a_{0s}.$$

Here B_i comes from the free electrons of the plasma, and the other terms come from the nonlinear susceptibilities $(\chi^{(2)},\chi^{(3)})$ of the bound electrons. Let

$$\varepsilon_i=g_ie^{\mathrm{i}\Omega_it+\mathrm{i}\alpha_i},\quad i=0,s,a,$$

and assume $\Omega_0=\Omega_s+\Omega_a$. We can get a set of equations for the real amplitudes g_i and real phases α_i (where $i=0,s,a$)

$$(1+\delta_0)\frac{\partial g_0}{\partial t}+\delta_{0I}\left(\frac{\omega_0}{2}-\Omega_0\right)g_0=(\beta_0\sin\gamma-\beta_{0I}\cos\gamma)g_sg_a-\frac{\partial\delta_0}{\partial t}g_0+\frac{2\pi\zeta_0}{N_0^2}\frac{\partial}{\partial t}(a_2g_sg_a\sin\gamma)-\frac{2\pi\zeta_0}{N_0^2}(a_1g_s^2g_0+a_2g_sg_a\cos\gamma)\frac{\partial\alpha_0}{\partial t},\tag{36}$$

$$(1+\delta_s)\frac{\partial g_s}{\partial t}+\delta_{sI}\left(\frac{\omega_s}{2}-\Omega_s\right)g_s=-(\beta_s\sin\gamma+\beta_{sI}\cos\gamma)g_0g_a-\frac{\partial\delta_s}{\partial t}g_s+\frac{2\pi\zeta_s}{N_0^2}\frac{\partial}{\partial t}(a_2g_0g_a\sin\gamma)+\frac{2\pi\zeta_s}{N_0^2}(a_1g_0^2g_s+a_2g_0g_a\cos\gamma)\frac{\partial\alpha_s}{\partial t},\tag{37}$$

$$\frac{\partial g_a}{\partial t}=\beta_0g_0g_s\sin\gamma,\tag{38}$$

$$(1+\delta_0)g_0\frac{\partial\alpha_0}{\partial t}-\delta_0\left(\frac{\omega_0}{2}-\Omega_0\right)g_0=(\beta_0\cos\gamma-\beta_{0I}\sin\gamma)g_sg_a+\frac{2\pi\zeta_0}{N_0^2}(a_2g_sg_a\sin\gamma)\times\frac{\partial\alpha_0}{\partial t}+\frac{2\pi\zeta_0}{N_0^2}(a_1g_s^2g_0+a_2g_sg_a\cos\gamma),\tag{39}$$

$$(1+\delta_s)g_s\frac{\partial\alpha_s}{\partial t}-\delta_s\left(\frac{\omega_s}{2}-\Omega_s\right)g_s=(\beta_s\cos\gamma-\beta_{sI}\sin\gamma)g_0g_a+\frac{2\pi\zeta_s}{N_0^2}(a_2g_0g_a\sin\gamma)\times\frac{\partial\alpha_s}{\partial t}-\frac{2\pi\zeta_s}{N_0^2}\frac{\partial}{\partial t}(a_1g_0^2g_s+a_2g_0g_a\cos\gamma),\tag{40}$$

$$g_a\frac{\partial\alpha_a}{\partial t}=-\beta_ag_0g_s\cos\gamma.\tag{41}$$

Here $\gamma=\alpha_0-\alpha_s-\alpha_a$ is the phase difference in SBS, and

$$\beta_{0I}=\frac{2\pi\zeta_0}{N_0^2}a_2\left(\Omega_0-\frac{\omega_0}{2}\right),\quad\delta_{0I}=-\frac{2\pi\zeta_0}{N_0^2}a_1g_s^2,$$

$$\beta_{sI}=-\frac{2\pi\zeta_s}{N_0^2}a_2\left(\Omega_s-\frac{\omega_s}{2}\right),\quad\delta_{sI}=\frac{2\pi\zeta_s}{N_0^2}a_1g_0^2.$$

When $\chi^{(2)}=\chi^{(3)}=0$, the above equations will transform to the conventional mode coupled equations of SBS in a full stripped plasma. Usually, Eqs. (35)–(41) can only be solved numerically.

B. Linear analyses of the SBS process

In the initial phase of SBS instability, we have $g_0(t)\simeq g_{00}=\mathrm{const}$, $g_s\sim g_a\sim 0\ll g_{00}$, $\partial\alpha_i/\partial t\sim 0$ $(i=0,s,a)$, $\gamma=\mathrm{const}$, and

$$\beta_{sI}^{(0)}\simeq\frac{\pi\zeta_s}{N_0^2}a_2\omega_s,\quad\delta_{sI}^{(0)}=\frac{2\pi\zeta_s}{N_0^2}a_1g_{00}^2,$$

$$\delta_s^{(0)}\simeq\frac{2\pi\chi^{(3)}}{N_0^2}(1+2a_{s0}^2)g_{00}^2.$$

The linearized evolution equations of g_s and g_a are

$$(1+\delta_s^0)\frac{\partial g_s}{\partial t}+\frac{1}{2}\delta_{sI}^2\omega_sg_s=-(\beta_s\sin\gamma+\beta_{sI}^{(0)}\cos\gamma)g_{00}g_a+\frac{2\pi\zeta_s}{N_0^2}a_2g_{00}\sin\gamma\frac{\partial g_a}{\partial t},\tag{42}$$

$$\frac{\partial g_a}{\partial t}=\beta_ag_{00}g_s\sin\gamma.$$

Let $g_i(t)=g_{i0}\exp(\Gamma t)(i=s,a)$. The growth rate of SBS instability can be derived from the linear algebraic equations

$$\Gamma=\frac{\Gamma_0}{\sqrt{1+\delta_s^{(0)}}}\left[\sqrt{1+\frac{\beta_{sI}}{\beta_s}\cot\gamma+\frac{(\delta_{sI}^{(0)}\omega_s)^2}{16(1+\delta_s^{(0)})\Gamma_0^2}}-\frac{\delta_{sI}^{(0)}\omega_s}{4\sqrt{1+\delta_s^{(0)}}\Gamma_0}\right].\tag{43}$$

Here

$$\Gamma_0=\sqrt{-\beta_s\beta_a}|\sin\gamma||\boldsymbol{e}_0\cdot\boldsymbol{e}_s|\equiv\frac{ek_ag_{00}}{4m\omega_0}\sqrt{\frac{\omega_{pi}^2\omega_0}{\omega_s^2\omega_a}}|\sin\gamma||\boldsymbol{e}_0\cdot\boldsymbol{e}_s|\tag{44}$$

is the conventional SBS instability growth rate when $\chi^{(2)}=\chi^{(3)}=0$.

When the linear terms of $\delta_i(\chi^{(3)})$ remain and for $\zeta_i(\chi^{(2)})=0$ (hence, the $\beta_{sI}=\delta_{sI}^{(0)}=0$), Eq. (43) becomes

$$\Gamma=\Gamma^{(0)}+\Gamma^{(3)}=\frac{\Gamma_0}{\sqrt{1+\delta_s^{(0)}}}. \quad (45)$$

Due to $\delta_s^{(0)}>0$, the third order susceptibility $\chi^{(3)}$ will always reduce the linear growth rate of SBS instability. According to Eqs. (25) and (26), the third nonlinear susceptibility $\chi^{(3)}$ can effect the linear instability of SBS only when the intensity of the pump laser $I_0>10^{17}$ W/cm^2. This is also the case in SRS.

Consider the effects of $\chi^{(2)}$ on the growth rate of SBS. Let

$$\zeta_s=\chi^{(2)}\xi_s, \quad \xi_s=(\boldsymbol{e}_\chi\cdot\boldsymbol{e}_0)(\boldsymbol{e}_s\cdot\boldsymbol{e}_a)+(\boldsymbol{e}_\chi\cdot\boldsymbol{e}_a)(\boldsymbol{e}_s\cdot\boldsymbol{e}_0),$$

where $\boldsymbol{e}_\chi=\boldsymbol{\chi}^{(2)}/\chi^{(2)}$. The first term of the $\chi^{(2)}$ in Eq. (43) becomes

$$\frac{\beta_{sI}}{\beta_s}=-\frac{4\pi\chi^{(2)}}{N_0^2}\frac{\xi_s}{a_{0s}}\frac{m_i}{e}\frac{\omega_s^2}{\omega_{pe}^2}k_aC_s^2$$

and the second term of the $\chi^{(2)}$ in Eq. (45) can be written as

$$\frac{\delta_{sI}\omega_s}{4\sqrt{1+\delta_s}\Gamma_0}\simeq\frac{\delta_{sI}\omega_s}{4\Gamma_0}=\frac{\pi\chi^{(2)}}{N_0^2}\frac{\xi_s}{|\sin\gamma||a_{0s}|}\sqrt{\frac{\omega_s^2\omega_a}{\omega_{pi}^2\omega_0}}g_{00}.$$

Though both terms are in proportion to the value of $\chi^{(2)}$, only the second term depends on the intensity of the pump wave. For estimating the values of the above terms, we will take the saturation (maximum) value of $\chi^{(2)}$ as $\chi^{(2)}\sim 5\times 10^{-8}$ cm/statvolt, and assume $\omega_0\sim\omega_s\sim\omega_{pe}\sim 10^{15}$ Hz, $\omega_{pi}^2\sim 10^{-3}\omega_{pe}^2\sim 10^{27}$ Hz, $\xi_s\sim a_{0s}$, $\sin\gamma\sim 1$, $T=T_i=3$ keV, $C_s\sim 3.5\times 10^7$ cm/s, $\omega_a\sim 4.4\times 10^{12}$ Hz $(k_a\sim k_0)$, then use $g_{00}=\sqrt{10^7(8\pi/c)I_0}$. Equation (45) becomes

$$\Gamma=\Gamma^{(0)}+\Gamma^{(2)}+\Gamma^{(3)}\simeq\frac{\Gamma_0}{\sqrt{1+\delta_s^{(0)}}}[\sqrt{1-0.34+2.1\times 10^{-16}I_0}-1.4\times 10^{-8}\sqrt{I_0}]. \quad (46)$$

From this expression, when the pump wave intensity $I_0>10^{14}$ W/cm^2, the second order nonlinear susceptibility $\chi^{(2)}$ will reduce the instability growth rate of SBS significantly. The pump wave intensity in the laser fusion experiments is usually $10^{14}<I_0<10^{16}$ W/cm^2, so the effect of $\chi^{(2)}$ on the instability of SBS is more observable than that of $\chi^{(3)}$ (the latter is active when $I_0>10^{17}$ W/cm^2).

From Eqs. (43), (45), and (46), we reach the following conclusions: (1) If we take the possible maximum value of $\chi^{(2)}$, the linear growth rate of SBS will be reduced enormously when the intensity of the pump laser $I_0>10^{14}$ W/cm^2. (2) If the frequency of the pump laser is far below any atomic resonance, the obvious effect of $\chi^{(3)}(N_2)$ on the linear growth rate of SBS occurs only when $I_0>10^{17}$ W/cm^2. In laser fusion, usually the intensity of the pump laser is $10^{14}<I_0<10^{16}$ W/cm^2, so the effect of $\chi^{(2)}$ on the SBS process may in this case be more important.

C. The nonlinear evolutions of the SBS

The numerical analysis of Eqs. (35)–(41) is similar to that for SRS. Consider a Au plasma as an example. The initial values are $g_{s0}/g_{00}=0.05$, $g_{a0}=n_0/(200Z)$(Z is the charge number of the Au ion), $\alpha_{00}=\alpha_{s0}=\alpha_{a0}=0$, $T_e=3$ keV, $n_i=10^{21}$cm^{-3}, $g_{00}=\sqrt{8\times 10^7\pi I_0/N_0}$, $\chi^{(1)}$ (esu) $=4\times 10^{-24}n_i$ (cm^{-3}), $\omega_0=6.3\times 10^{15}$ s^{-1}, $\omega_s\simeq\omega_0$, $\alpha_{0s}=\pi/4=\alpha_{0a}$, $\alpha_{sa}=\pi/2$, $\boldsymbol{\chi}^{(2)}\cdot\boldsymbol{e}_0=\chi^{(2)}$, $\boldsymbol{\chi}^{(2)}\cdot\boldsymbol{e}_s=\chi^{(2)}\cos(\pi/4)=\boldsymbol{\chi}^{(2)}\cdot\boldsymbol{e}_a$. Shown in Figs. 3(a)–3(d) are results of numerical solutions. Just similar to Fig. 1, Fig. 3(a) shows the general behavior of the SBS nonlinear evolution.

(a) *Pure effects of the third susceptibility* $\chi^{(3)}(\chi^{(2)}=0)$. Figure 3(b) shows the effect of $\chi^{(3)}$ on the SBS while the pump strength (hence amplitude) is constant. The growth rate of scattered waves decreases and the saturation time lengthens with increasing $\chi^{(3)}$, but the saturation amplitude of the scattering wave remains almost constant at any value of $\chi^{(3)}$. In Fig. 3(b), the strength of pump wave is 10^{16} W/cm^2, and the effect of $\chi^{(3)}$ is large, which illustrates the sensitivity of $\chi^{(3)}$.

(b) *Effects of the second order nonlinear susceptibility* $\chi^{(2)}(\chi^{(3)}=0)$. Figure 3(c) shows the effect of $\chi^{(2)}$ on SBS evolution for $I_0=10^{16}$ W/cm^2. The growth rate decreases with $\chi^{(2)}$, while the saturation amplitude of the scattering wave decreases only slightly. From the above results, we can also find that the effects of $\chi^{(2)}$ and $\chi^{(3)}$ on the SBS process are similar, which all act as a stabilizing factor. This is different from the SRS case, in which $\chi^{(2)}$ destabilizes the SRS while $\chi^{(3)}$ stabilizes the SRS.

(c) *Effects of both second and third order nonlinear susceptibilities* $(\chi^{(2)},\chi^{(3)}\neq 0)$. The effects of both $\chi^{(2)}$ and $\chi^{(3)}$ on the SBS process are shown in Fig. 3(d). When $\chi^{(3)}$ and $\chi^{(2)}$) are fixed, we find that the initial pump wave strength also has a large effect on SBS. With increasing I_0 , the relative saturation amplitudes and growth rate of the scattering wave all decrease monotonically. There is also a critical strength $I_{0c}\simeq 4\times 10^{15}$ W/cm^2, above which the saturation time of the scattering wave decreases with increasing I_0. When $I<I_{0c}$, the saturation time of the scattering wave monotonically increases with increasing I_0.

V. CONCLUSION

The nonlinear evolutions of pump and scattering waves, which are periodic (and not damped due to the absent of dissipation mechanism in our model), can be characterized qualitatively by three factors: the growth rate, the saturation time, and the saturation amplitude. In SRS, the second order susceptibility $\chi^{(2)}$ will destabilize the SRS process, increasing the growth rate of instability while reducing the saturation time and the saturation amplitude. The third order susceptibility $\chi^{(3)}$ is found to stabilize the SRS process. In SBS, both $\chi^{(2)}$ and $\chi^{(3)}$ stabilize the SBS process. Increasing the pump wave strength I_0 can destabilize or stabilize the SRS and SBS processes, depending on the values of $\chi^{(2)}$ and $\chi^{(3)}$. Choosing the proper material to form a plasma with certain values of $\chi^{(2)}$ and $\chi^{(3)}$ may therefore avoid instabilities of

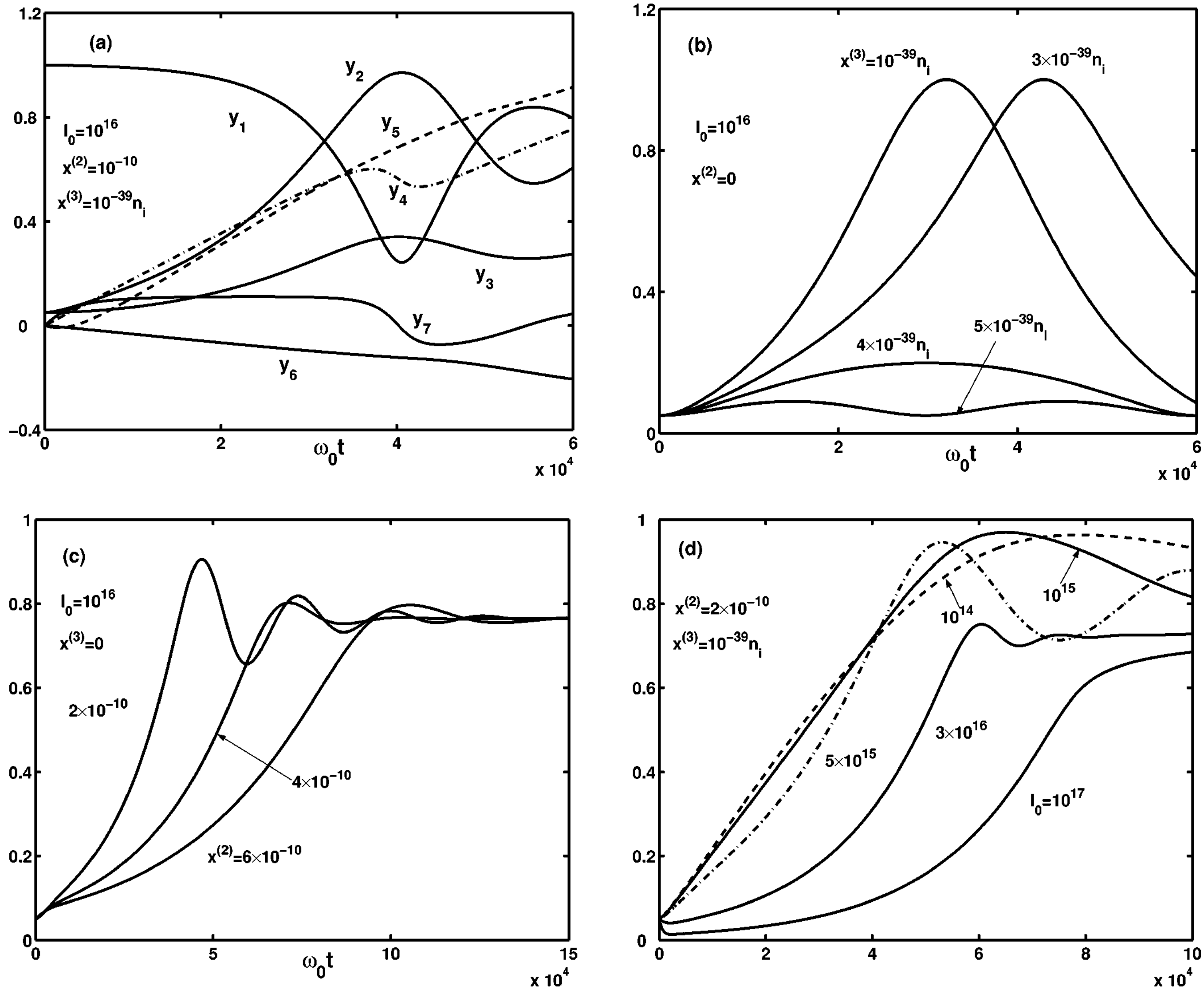

FIG. 3. (a) General behavior of SBS nonlinear evolution, y_7 is similar to Fig. 1. (b)–(d) The relative amplitudes of the scattering electromagnetic wave evolves with time when two of I_0 (W/cm^2), $\chi^{(2)}$ (esu), and $\chi^{(3)}$ (esu) are constant and another is various in SBS.

SRS and SBS which can potentially degrade the laser coupling in ICF geometries, for example. Unfortunately, accurate values of nonlinear susceptibilities in partially stripped plasmas are still not well known. In this paper the values of $\chi^{(2)}$ and $\chi^{(3)}$ are approximately taken according to Ref. [1]. The values of nonlinear susceptibilities and the functional dependence of polarization on the electric field of the pump and plasma waves should also be further studied.

[1] P. Sprangle, E. Esarey, and B. Hafizi, Phys. Rev. Lett. **79**, 1046 (1997).
[2] P. Sprangle, E. Esarey, and B. Hafizi, Phys. Rev. E **56**, 5894 (1997).
[3] R. W. Boyd, *Nonlinear Optics* (Academic Press, Boston, 1992), Chap. 1.
[4] William L. Kruer, *The Physics of Laser Plasma Interactions* (Addison-Wesley, Reading, MA, 1988), Chap. 6.
[5] Hu Xi-wei, Chin. Phys. Lett. 13, 28 (1996).
[6] Wang Shan-cai and Hu Xi-wei, Chin. Phys. Lett. **14**, 678 (1997).
[7] Hu Xi-wei, Chin. Phys. Lett. **17**, 893 (2000).
[8] Hu Xi-wei, Plasma Sources Sci. Technol. **2**, 279 (2000).